Exercise 1

Economic growth: Theory and Empirical Methods, UC3M

Question 1: Consider the following functions:

$$x(t) = x(0)\exp(0.02t) \tag{1}$$
$$y(t) = y(0)\exp(0.05t) \tag{2}$$

1. Compute the growth rate of $x(t)$.
2. Compute the growth rate of $y(t)$.
3. Compute the growth rate of $x(t)y(t)$.
4. Compute the growth rate of $\frac{x(t)}{y(t)}$.

Question 2: Consider the function:

$$y = (kl)^{\beta}\left(\frac{1}{m}\right)^{1-\beta} \tag{3}$$

Calculate the growth rate of y as a function of the growth rates of k, l, and m.

Question 3: Open the R file *students*1. The file provides the value of y in period 0, the length of the simulation period, T, and the year of each simulation period, *time*.

1. For an exponential growth rate of $g = 0.05$, compute y in each period $y(t) = y(0)\exp(gt)$.
2. For a linear growth rate of $g_L = 0.05$, compute y_L in each period $y_L(t) = y(0) + tg_L$.
3. For a geometric growth rate of $g_g = 0.05$, compute y_g in each period $y_g(t) = y(0)(1 + g_g)^t$.
4. On the vector $y(t)$, find the first period in which output has doubled using the "which" function from R.

Question 4:

1. Consider the Malthus model. In R, plot labor growth on the y-axis against labor on the x-axis. To this end, use $A = 50$, $\alpha = 0.5$, and $Z = 0.1$. To create the x-axis, use the "seq" command from R to create a linear-spaced vector with 100 points between $[0.5L^*, 2L^*]$.
2. Plot into the same graph what happens to the labor growth schedule when the fixed factor increases by 30%.

3. Assume $A = 50$ and, in period 0, we have output per worker $y = 10$ and the amount of labor is $L = 25$. Suppose that because of the black death, labor falls to $L = 10$. Plot output per worker for the next 100 years. To that end, use a "for loop" and compute for each period

   - the labor growth rate: $n(t) = Z - \frac{1}{y(t)}$.
   - labor next period: $L(t+1) = (1 + n(t))L(t)$.
   - the resulting output per worker.

4. Assume instead that the plague not only decreases labor but also decreases the fixed factor by decreasing productivity: $A = 25$. Redraw output per worker over time.

5. Use the solution to the differential equation (Equation 25) to plot for 100 periods the behavior of $y(t)$ for $y(0) = 80$.